

Introduction

Please allow us to introduce you to Ashley Norris, Joanna Halstrom, Luciana Fernandez, and Jasvinder Singh. These four middle school mathematics teachers set out each day to deliberately, intentionally, and purposefully impact the learning of their students. Whether they teach sixth, seventh, or eighth grade, they recognize that

- They have the capacity to select and implement various teaching and learning strategies that enhance their adolescent learners in mathematics;
- The decisions they make about their teaching have an impact on their students' learning;
- Each and every student can learn mathematics and they take responsibility to teach *all* learners in their middle school mathematics classes; and
- They must continuously question and monitor the impact of their teaching on student learning. (adapted from Hattie & Zierer, 2018)

Through the videos accompanying this book, you will meet three additional middle school mathematics teachers and the instructional leaders that support them in their teaching. Collectively, the recognitions above—or their **mindframes**—lead to action in their mathematics classrooms and their actions lead to outcomes in student learning. This is where we begin our journey through *Teaching Mathematics in the Visible Learning Classroom*.

Visible Learning occurs when teachers *see* learning through the eyes of their students and students *see* themselves as their own teachers. How do teachers of mathematics see integers, rational numbers, absolute

Mindframes are ways of thinking about teaching and learning. Teachers who possess certain ways of thinking have major impacts on student learning.

value, proportional relationships, functions, and linear equations through the eyes of their students? In turn, how do teachers develop assessment-capable visible learners—students who see themselves as their own teachers—in the study of numbers, operations, and relationships? Conceptualizing, implementing, and sustaining Visible Learning in the middle school mathematics classroom by identifying *what works best* and *what works best when* is exactly what we set out to do in this book.

Mathematics learning involves the balance of conceptual understanding, procedural knowledge, and the application of concepts and thinking skills to a variety of mathematical contexts. By balance, we mean that no one dimension of mathematics learning is more important than the other two. Conceptual understanding, procedural knowledge, and the application of concepts and thinking skills are each essential aspects of learning mathematics. Mathematics classrooms where *teachers see learning through the eyes of their learners and learners see themselves as their own teachers* result from specific, intentional, and purposeful decisions about each of these dimensions of mathematics instruction critical for student growth and achievement. This book explores each of these components in middle school mathematics teaching and learning through the lens of *what works best* in student learning at the surface, deep, and transfer phases. We are not suggesting that teachers implement procedural knowledge, conceptual understanding, and application in isolation, but through a series of linked learning experiences and challenging mathematical tasks that result in students engaging in both mathematical content and processes.

Our Learning Intention: To understand what works best in the middle school mathematics classroom.

What Works Best

Identifying what works best draws from the key findings from Visible Learning (Hattie, 2009) and also guides the classrooms described in this book. One of those key findings is that *there is no one way to teach mathematics or one best instructional strategy that works in all situations for all students*, but there is compelling evidence for certain strategies and approaches that have a greater likelihood of helping students reach their learning goals. In this book, we use the effect size information that John Hattie has collected and analyzed over many years to inform how we

transform the findings from the Visible Learning research into learning experiences and challenging mathematical tasks that are most likely to have the strongest influence on student learning.

For readers less familiar with Visible Learning, we would like to take a moment to review what we mean by *what works best*. The Visible Learning database is composed of over 1,800 meta-analyses of studies that include over 80,000 studies and 300 million students. Some have argued that it is the largest educational research database amassed to date. To make sense of so much data, John Hattie focused his work on meta-analyses. A **meta-analysis** is a statistical tool for combining findings from different studies, with the goal of identifying patterns that can inform practice. In other words, a meta-analysis is a study of studies. The mathematical tool that aggregates the information is an effect size, and can be represented by Cohen's *d*. An **effect size** is the magnitude, or relative size, of a given effect. Effect size information helps readers understand not only that something does or does not have an influence on learning but also the relative impact of that influence.

For example, imagine a hypothetical study in which learning mathematics while sitting on a stability ball results in relatively higher mathematics scores. Schools and classrooms around the country might devote large monetary resources to removing traditional chairs and desks, replacing this furniture with stability balls for all of the middle school mathematics classrooms. However, let's say the results of this hypothetical study also indicate that the "stability ball effect" had an effect size of 0.03 in mathematics achievement over the control group, an effect size pretty close to zero. Furthermore, the large number of students participating in the study made it almost certain there would be a difference in the two groups of students (those using a stability ball versus those not using a stability ball). As an administrator or teacher, would you still advocate for spending large amounts of your district or school budget on non-traditional seating? How confident would you be in the impact or influence of your decision on mathematics achievement in your district or school?

This is where an effect size of 0.03 for the "stability ball effect" is helpful. Understanding the effect size helps us know how powerful a given influence is in changing achievement—in other words, the impact for the effort or return on the investment. The effect size helps us understand

A **meta-analysis** is a statistical tool for combining findings from different studies, with the goal of identifying patterns that can inform practice.

Effect size represents the magnitude of the impact that a given approach has.



Video 1 What Is Visible Learning for Mathematics?

To read a QR code, you must have a smartphone or tablet with a camera. We recommend that you download a QR code reader app that is made specifically for your phone or tablet brand. Videos can also be accessed at <https://resources.corwin.com/vlmathematics-6-8>

not just what works, but *what works best*. With the increased frequency and intensity of mathematics initiatives, programs, and packaged curricula, deciphering where to best invest resources and time to achieve the greatest learning outcomes for all students is challenging and frustrating. For example, some programs or packaged curricula are hard to implement and have very little impact on student learning, whereas others are easy to implement but still have limited influence on student growth and achievement in mathematics. Teaching mathematics in the Visible Learning classroom involves searching for those things that have the greatest impact and produce the greatest gains in learning, some of which will be harder to implement and some of which will be easier to implement.

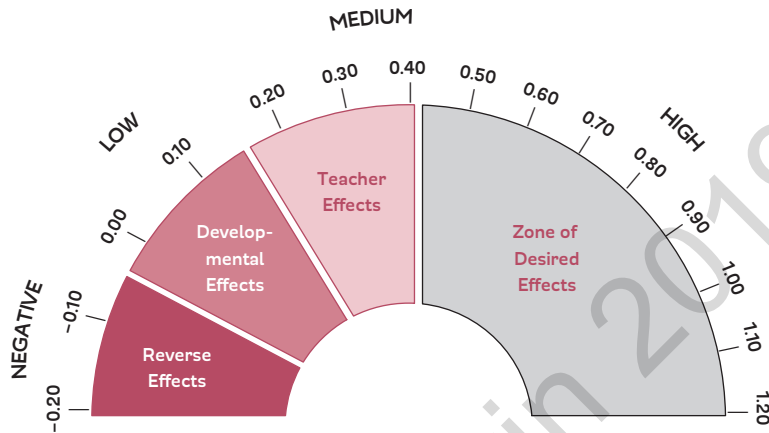
As we begin planning for our first period Math 6 class or our afternoon Math 8 class, knowing the effect size of different influences, strategies, actions, and approaches to teaching and learning proves helpful in deciding where to devote our planning time and resources. Is a particular approach (e.g., classroom discussion, exit tickets, use of calculators, a jigsaw activity, computer-assisted instruction, simulation creation, cooperative learning, instructional technology, presentation of clear success criteria, development of a rubric, etc.) worth the effort for the desired learning outcomes of that day, week, or unit? With the average effect size across all influences measuring 0.40, John Hattie was able to demonstrate that influences, strategies, actions, and approaches with an effect size greater than 0.40 allow students to learn at an appropriate rate, meaning at least a year of growth for a year in school. Effect sizes greater than 0.40 mean more than a year of growth for a year in school. Figure I.1 provides a visual representation of the range of effect sizes calculated in the Visible Learning research.

Before this level was established, teachers and researchers did not have a way to determine an acceptable threshold, and thus we continued to use weak practices, often supported by studies with statistically significant findings.

Consider the following examples. First, let us consider classroom discussion. Should teachers devote resources and time to planning for the facilitation of classroom discussion? Will this approach to mathematics provide a return on investment, rather than “chalk talk” where we work out lots of problems on the board for students to include in their notes?

EFFECT SIZE FOR
ABILITY GROUPING
= 0.12

THE BAROMETER OF INFLUENCE



Source: Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement*. Figure 2.4, page 19. New York, NY: Routledge.

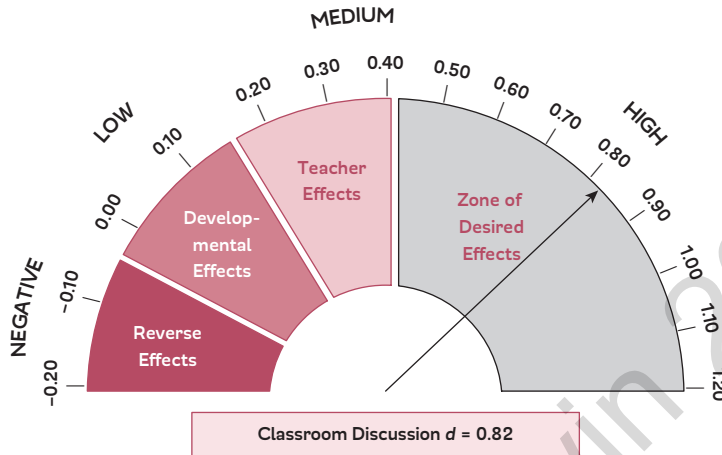
Figure I.1

With classroom discussion, teachers intentionally design and purposefully plan for learners to talk with their peers about specific problems or approaches to problems (e.g., demonstrating multiple representations of operations with fractions or comparing and contrasting dependent and independent events) in collaborative groups. Peer groups might engage in working to solve complex problems or tasks (e.g., determining congruence or graphing inequalities on a number line). The students would not be **ability grouped**, but rather grouped by the teacher to ensure that there is academic diversity in each group as well as language support and varying degrees of interest and motivation. As can be seen in the barometer in Figure I.2, the effect size of classroom discussion is 0.82, which is well above our threshold and is likely to accelerate learning gains.

Therefore, individuals teaching mathematics in the Visible Learning classroom would use classroom discussion to understand mathematics learning through the eyes of their students and for students to see themselves as their own mathematics teachers.

Ability grouping, also referred to as tracking or streaming, is the long-term grouping or tracking of learners based on their ability. This is different from flexibly grouping students to work on a specific concept, skill, or application or address a misconception.

THE BAROMETER FOR THE INFLUENCE OF CLASSROOM DISCUSSION



Source: Adapted from Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement*. Figure 2.4, page 19. New York, NY: Routledge.

Figure I.2

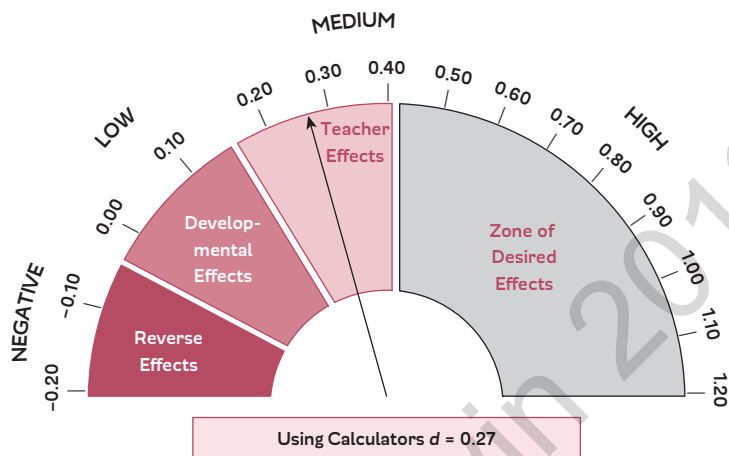
EFFECT SIZE FOR CLASSROOM DISCUSSION = 0.82

EFFECT SIZE OF USE OF CALCULATORS = 0.27

Second, let us look at the use of calculators. Within academic circles, teacher workrooms, school hallways, and classrooms, there have been many conversations about the use of the calculator in mathematics. There have been many efforts to reduce the reliance on calculators while at the same time developing technology-enhanced items on assessments in mathematics. Using a barometer as a visual representation of effect sizes, we see that the use of calculators has an overall effect size of 0.27. The barometer for the use of calculators is shown in Figure I.3.

As you can see, the effect size of 0.27 is below the zone of desired effects of 0.40. The evidence suggests that the impact of the use of calculators on mathematics achievement is low. However, closer examination of the five meta-analyses and the 222 studies that produced an overall effect size of 0.27 reveals a deeper story to the use of calculators. Calculators are most effective when they are used (1) for computation, deliberate practice, or learners checking their work; (2) to reduce the amount of cognitive load on learners as they engage in problem

THE BAROMETER FOR THE INFLUENCE OF USING CALCULATORS



Source: Adapted from Hattie, J. (2009). *Visible learning: A synthesis of over 800 meta-analyses relating to achievement*. Figure 2.4, page 19. New York, NY: Routledge.

Figure 1.3

solving; and (3) when there is an intention behind using them (e.g., solving by graphing or approximation problems). This leads us into a second key finding from John Hattie's Visible Learning research: *We should not hold any influence, instructional strategy, action, or approach to teaching and learning in higher esteem than students' learning.*

What Works Best When

Visible learning in the mathematics classroom is a continual evaluation of our impact on student learning. From the above example, the use of calculators is not really the issue and should not be our focus. Instead, our focus should be on the intended learning outcomes for that day and how calculators support that learning. Visible Learning is more than a checklist of dos and don'ts. Rather than checking influences with high effect sizes off the list and scratching out influences with low effect sizes, we should match the *best* strategy, action, or approach with learning needs of our students. In other words, is use of calculators the right strategy

Teaching Takeaway

Using the right approach at the right time increases our impact on student learning in the mathematics classroom.

or approach for the learners at the right time, for this specific content? Clarity about the learning intention brings into focus what the learning is for the day, why students are learning about this particular piece of content and process, and how we *and* our learners will know they have learned the content. Teaching mathematics in the Visible Learning classroom is not about a specific strategy, but a location in the learning process.

Over the next several chapters, we will show how to support mathematics learners in their pursuit of conceptual understanding, procedural knowledge, and application of concepts and thinking skills through the lens of *what works best when*. This requires us, as mathematics teachers, to be clear in our planning and preparation for each learning experience and challenging mathematics tasks. Using the guiding questions in Figure I.5 (on pages 12–13), we will model how to blend what works best with what works best *when*. You can use Figure I.5 in your own planning. This planning guide is found also in Appendix B and online at resources.corwin.com/vlmathematics-6-8.

EFFECT SIZE FOR
ASSESSMENT-
CAPABLE VISIBLE
LEARNERS = 1.33

Through these specific, intentional, and purposeful decisions in our mathematics instruction, we pave the way for helping learners see themselves as their own teachers, thus making them assessment-capable visible learners in mathematics.

The Path to Assessment-Capable Visible Learners in Mathematics

Teaching mathematics in the Visible Learning classroom builds and supports assessment-capable visible learners (Frey, Hattie, & Fisher, 2018). With an effect size of 1.33, providing a mathematics learning environment that allows learners to see themselves as their own teacher is essential in today's classrooms.

Assessment-capable visible mathematics learners are

1. Active in their mathematics learning. Learners deliberately and intentionally engage in learning mathematics content and processes by asking themselves questions, monitoring their own learning, and taking the reins of their learning. They know their current level of learning.

An assessment-capable visible learner says, "I am comfortable finding the area of a circle but need more learning on surface area and volume. I know there are examples in my interactive notebook that I can use to prepare for tomorrow's challenge problem."

2. Able to plan the immediate next steps in their mathematics learning within a given unit of study or topic. Because of the active role taken by an assessment-capable visible mathematics learner, these students can plan their next steps and select the right tools (e.g., manipulatives, problem-solving approaches, and/or meta-cognitive strategies) to guide their learning. They know what additional tools they need to successfully move forward in a task or topic.

An assessment-capable visible learner says, "To find the volume of this cylinder, I am going to calculate the radius of the circle rather than measure it. This makes my volume calculation more precise."

3. Aware of the purpose of the assessment and feedback provided by peers and the teacher. Whether the assessment is informal, formal, formative, or summative, assessment-capable visible mathematics learners have a firm understanding of the information behind each assessment and the feedback exchanged in the classroom. Put differently, these learners not only seek feedback but also recognize that errors are opportunities for learning, monitor their progress, and adjust their learning. (adapted from Frey et al., 2018)

An assessment-capable visible learner says, "Yesterday's exit ticket surprised me. Ms. Norris wrote on my paper that I needed to revisit the process for isolating the unknown variable. So, today, I am going to work out the entire process in my notebook and not try and do it all in my head."

Over the next several chapters, we will explore how to create a classroom environment that focuses on learning and provides the best environment for developing assessment-capable visible mathematics learners who can engage in the mathematical habits of mind represented in one form or another in every standards document. Such learners can

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

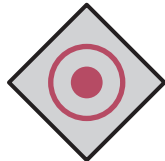


Video 2
Creating Assessment-Capable Visible Learners

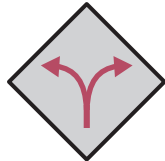
<https://resources.corwin.com/vlmathematics-6-8>

ASSESSMENT-CAPABLE VISIBLE LEARNERS

ASSESSMENT-CAPABLE LEARNERS:



KNOW THEIR CURRENT LEVEL OF UNDERSTANDING



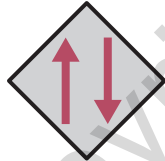
KNOW WHERE THEY'RE GOING AND ARE CONFIDENT TO TAKE ON THE CHALLENGE



SELECT TOOLS TO GUIDE THEIR LEARNING



SEEK FEEDBACK AND RECOGNIZE THAT ERRORS ARE OPPORTUNITIES TO LEARN



MONITOR THEIR PROGRESS AND ADJUST THEIR LEARNING



RECOGNIZE THEIR LEARNING AND TEACH OTHERS

Source: Adapted from Frey, Hattie, & Fisher (2018).

Figure I.4

4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning (© Copyright 2010. National Governors Association Center for Best Practices and Council of Chief State School Officers. All rights reserved.).

How This Book Works

As authors, we assume you have read *Visible Learning for Mathematics* (Hattie, Fisher, Frey, Gojak, Moore, & Mellman, 2017), so we are not going to recount all of the information contained in that book. Rather, we are going to dive deeper into aspects of middle school mathematics instruction that are critical for students' success, helping you to envision what a Visible Learning mathematics classroom like yours looks like. In each chapter, we profile three middle school teachers who have worked to make mathematics learning visible for their students and have influenced learning in significant ways. Each chapter will do the following:

1. Provide effect sizes for specific influences, strategies, actions, and approaches to teaching and learning.
2. Provide support for specific strategies and approaches to teaching mathematics.
3. Incorporate content-specific examples from middle school mathematics curricula.
4. Highlight aspects of assessment-capable visible learners.

Through the eyes of sixth, seventh, and eighth grade mathematics teachers, as well as the additional middle school mathematics teachers and the instructional leaders in the accompanying videos, we aim to show you the mix and match of strategies you can use to orchestrate your lessons in order to help your students build their conceptual understanding, procedural fluency, and application of concepts and thinking

HOW TO USE APPENDIX B WHEN PLANNING FOR CLARITY

I have to be clear about what content and practice or process standards I am using to plan for clarity. Am I using only mathematics standards or am I integrating other content standards (e.g., writing, reading, or science)?

Rather than what I want my students to be doing, this question focuses on the learning. What do the standards say my students should learn? The answer to this question generates the **learning intentions** for this particular content.

Once I have clear learning intentions, I must decide when and how to communicate them with my learners. Where does it best fit in the instructional block to introduce the day's learning intentions? Am I going to use guiding questions?

As I gather evidence about my students' learning progress, I need to establish what they should know, understand, and be able to do that would demonstrate to me that they have learned the content. This list of evidence generates the **success criteria** for the learning.

ESTABLISHING PURPOSE

1

What are the key content standards I will focus on in this lesson?

Content Standards:

2

What are the learning intentions (the goal and *why* of learning, stated in student-friendly language) I will focus on in this lesson?

Content:

Language:

Social:

3

When will I introduce and reinforce the learning intention(s) so that students understand it, see the relevance, connect it to previous learning, and can clearly communicate it themselves?

SUCCESS CRITERIA

4

What evidence shows that students have mastered the learning intention(s)? What criteria will I use?

I can statements:



This planning guide is available for download at resources.corwin.com/vlmathematics-6-8.

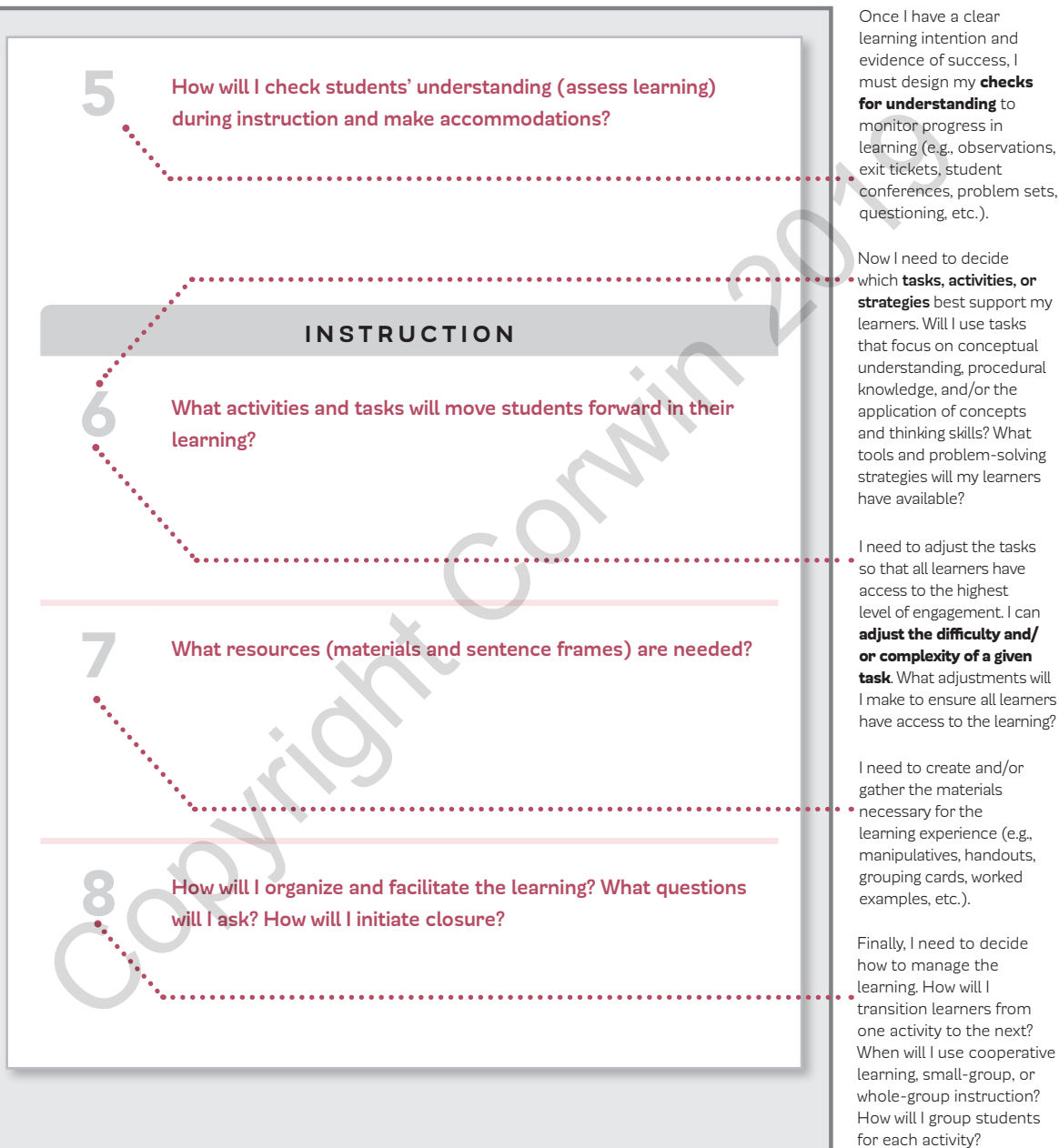


Figure I.5

skills in the most visible ways possible—visible to you and to them. If you're a mathematics specialist, mathematics coordinator, or methods instructor, you may be interested in exploring the vertical progression of these content areas PreK–12 within Visible Learning classrooms and see how visible learners grow and progress across time and content areas. Although you may identify with one of the teachers from a content perspective, we encourage you to read all of the vignettes to get a full sense of the variety of choices you can make in your instruction, based on your instructional goals.

In Chapter 1, we focus on the aspects of mathematics instruction that must be included in each lesson. We explore the components of effective mathematics instruction (conceptual, procedural, and application) and note that there is a need to recognize that student learning has to occur at the surface, deep, and transfer levels within each of these components. Surface, deep, and transfer learning served as the organizing feature of *Visible Learning for Mathematics*, and we will briefly review them and their value in learning. This book focuses on the ways in which teachers can develop students' surface, deep, and transfer learning, specifically by supporting students' conceptual understanding, procedural knowledge, and application whether with linear equations or integer operations. Finally, Chapter 1 contains information about the use of checks for understanding to monitor student learning. Generating evidence of learning is important for both teachers and students in determining the impact of the learning experiences and challenging mathematical tasks on learning. If learning is not happening, then we must make adjustments.

Following this introductory chapter, we turn our attention, separately, to each component of mathematics teaching and learning. However, we will walk through the process starting with the application of concepts and thinking skills, then direct our attention to conceptual understanding, and finally, procedural knowledge. This seemingly unconventional approach will allow us to start by making the goal or endgame visible: learners applying mathematics concepts and thinking skills to other situations or contexts.

Chapter 2 focuses on *application* of concepts and thinking skills. Returning to our three profiled classrooms, we will look at how we plan,

develop, and implement challenging mathematical tasks that scaffold students' thinking as they apply their learning to new contexts or situations. Teaching mathematics in the Visible Learning classroom means supporting learners as they use mathematics in a variety of situations. In order for learners to effectively apply mathematical concepts and thinking skills to different situations, they must have strong conceptual understanding and procedural knowledge. Returning to Figure I.5, we will walk through the process for establishing clear learning intentions, defining evidence of learning, and developing challenging tasks that, as you already have come to expect, encourage learners to see themselves as their own teachers. Each chapter will discuss how to differentiate mathematical tasks by adjusting their difficulty and/or complexity, working to meet the needs of all learners in the mathematics classroom.

Chapters 3 and 4 take a similar approach with conceptual understanding and procedural knowledge, respectively. Using Chapter 2 as a reference point, we will return to the three profiled classrooms and explore the conceptual understanding and procedural knowledge that provided the foundation for their learners applying ideas to different mathematical situations. For example, what influences, strategies, actions, and approaches support a learner's conceptual understanding of systems of equations, measurement of circles, or integer operations? With conceptual understanding, what works best as we encourage learners to see mathematics as more than a set of mnemonics and procedures? Supporting students' thinking as they focus on underlying conceptual principles and properties, rather than relying on memory cues like PEMDAS, also necessitates adjusting the difficulty and complexity of mathematics tasks. As in Chapter 2, we will talk about differentiating tasks by adjusting their difficulty and complexity.

In this book, we do not want to discourage the value of procedural knowledge. Although mathematics is more than procedural knowledge, developing skills in basic procedures is needed for later work in each area of mathematics from the area and circumference of a circle to linear equations. As in the previous two chapters, Chapter 4 will look at what works best when in supporting students' fluency in procedural knowledge. Adjusting the difficulty and complexity of tasks will once again help us meet the needs of all learners.

In the final chapter of this book, we focus on how to make mathematics learning visible through evaluation. Teachers must have clear knowledge of their impact so that they can adjust the learning environment. Learners must have clear knowledge about their own learning so that they can be active in the learning process, plan the next steps, and understand what is behind the assessment. What does evaluation look like so that teachers can use it to plan instruction and to determine the impact that they have on learning? As part of Chapter 5, we highlight the value of feedback and explore the ways in which teachers can provide effective feedback to students that is growth producing. Furthermore, we will highlight how learners can engage in self-regulation feedback and provide feedback to their peers.

This book contains information on critical aspects of middle school mathematics instruction that have evidence for their ability to influence student learning. We're not suggesting that these be implemented in isolation but rather that they be combined into a series of linked learning experiences that result in students engaging in mathematics learning more fully and deliberately than they did before. Whether calculating slope or the area of a circle, we strive to create a mathematics classroom where we *see* learning through the eyes of our students and students *see* themselves as their own mathematics teachers. As learners progress from simplifying rational expressions to using ratios and proportions, teaching mathematics in the Visible Learning classroom should build and support assessment-capable visible mathematics learners.